

# 2D Shape Matching based on B-spline Curves and Dynamic Programming

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**Abstract:** In this paper, we propose an approach for two-dimensional shape representation and matching using the B-spline modelling and Dynamic Programming (DP), which is robust with respect to affine transformations such as translation, rotation, scale change and some distortions. Boundary shape is first split into distinct parts based on the curvature. Curvature points are critical attributes for shape description, allowing the concave and convex parts of an object representation, which are obtained by the polygonal approximation algorithm in our approach. After that each part is approximated by a normalized B-spline curve using some global features including the arc length, the centroid of the shape and moments. Finally, matching and retrieval of similar shapes are obtained using a similarity measure defined on their normalized curves with Dynamic Programming. Dynamic programming not only recovers the best matching, but also identifies the most similar boundary parts. The experimental results on some benchmark databases validate the proposed approach.

## 1 INTRODUCTION

One of the most popular image information in computer vision is the shape. The object shape provides a powerful visual feature for shape representation, recognition, matching, classification..

Many approaches have been proposed for shape modelling; most of them focus either on shape boundaries or on interior region of shape. Region-based methods which are easy to compute take into account global information such as: area, circularity and Fourier descriptor (Zhang and Lu, 2002).

Moments based shape descriptors are the most popular region-based methods (Kim and Kim, 2000). There are different shape moments such as the geometric moments, Legendre moments (Yang et al, 2006) and Zernike moments (Singh and Pooja, 2011). Although these methods achieve reasonable results, they are not robust in case of occlusion and do not allow partial matching (Dao and De Amicis, 2006).

On the other hand, boundary-based methods which use curvature focus on the extraction of features from the boundary contour. Fourier descriptor (Zhang and Lu, 2002), chain codes (Dubois and Glanz, 1986) and wavelet descriptors

are some of the effective boundary-based shape methods. Curvature scale space (Mokhtarian et al., 1996) is a rich descriptor which represents the shape curve by convolving the curve with a Gaussian function at different scales and extracts the inflexion points along the resulting curves.

In shape context descriptor (Belongie et al., 2002), the authors describe a shape as a set of sample points with the geometric relationship between them. A shape context at a sample point captures the distribution of the rest points relatively to it.

Another shape descriptor is the Medial Axis Transform, which is presented by Blum (Blum, 1967) and later Sebastian and al (Sebastian et al, 2004) used this descriptor for shape recognition.

In the literature, the notion of a part-based representation has played an important role in object recognition. For example, in (Latecki and Lakamper, 2000), the authors used a discrete curve evolution technique to decompose a boundary shape into parts. Then a shape similarity measure based on the correspondence of visual parts is defined in order to achieve the matching of two shapes.

In another work (Alajlan et al., 2007), the proposed descriptor is based on triangle area

representation (TAR) to measure concavity and convexity of each boundary point at multi scales.

Daliri and al in (Daliri et al., 2010) proposed a representation for shape-based recognition based on the extraction of the perceptually relevant fragments. According to this method, each shape is described by a set of symbols based on the extracted segments which is mapped to an invariant dimensional space that is used for recognition.

Other techniques consist of approximate the shape contour by the polygonal approximation (Arkin et al., 1991; Carmona-poyato et al., 2010), B-spline (Paglieroni, 1985), and height functions (Wang et al., 2012).

The B-splines possess attractive properties such as continuity, smoothness and affine transformation invariance that make them suitable for shape representation. In (Cohen et al., 1995), the authors used the B-splines to represent and match 2D objects like handwriting and aircrafts. Wang and Teoh (Wang and Teoh, 2004) consider the B-splines curves and their curvature scale space for 2D shape matching algorithm. In another work (Mongkolnam et al., 2007) propose a technique for representing structural features of images based on B-splines curves and chain code.

In the above references, the B-splines are used to extract features from boundary or to curve representation. However, few works have used the B-spline representation in a 2D image analysis (Mongkolnam et al., 2007).

In this paper we propose a recognition system which is invariant to translation, rotation, scale change and small amount of deformation. After decomposing 2D objects into meaningful parts, curve normalization based on the B-splines model and invariant moments are applied in order to ensure the affine invariant shape representation. The matching algorithm that follows matches the obtained curves using the dynamic programming (DP). DP selects among all possible matching curves the most promising one with the minimal distance. Two shapes are considered similar when the cost with both shape representations is minimal.

Our contributions of this work are:

- First, we propose a part-based method for shape representation based on the curvature points and normalized curves.
- Second, we propose to explore directly the obtained curves to matching and retrieving.

## 2 PROPOSED APPROACH

In our approach to represent 2D shape we have to segment the contour shape into elementary parts. The segmented boundaries are first modelled by B-spline curves. Then, the obtained curves are normalized in several steps in order to eliminate translation, scaling and rotation transformations. So some global features are associated to a shape  $S$  as the centroid of shape, the minimum area rectangle and moments.

- The centroid of the shape  $(x_G, y_G)$  is normalized so as to coincide with the origin. It is defined by the first moment order as:

$$x_G = M_{10} / M_{00}, \quad y_G = M_{01} / M_{00} \quad (1)$$

where

$$M_{pq} = \sum_i \sum_j i^p j^q f(i, j) \quad (2)$$

and the intensity function  $f$ :

$$f(i, j) = 1, \quad \forall (i, j) \in S. \quad (3)$$

- The minimum area rectangle enclosing a silhouette is defined as the smallest rectangle minimizing the area between the shape and its convex hull (Philip et al., 2002). It is unique for each shape and it is invariant to rotation. In our approach, the shape is reoriented so that the width of the rectangle is parallel to the Y axis.

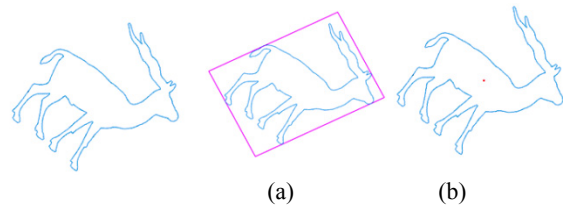


Figure 1: (a) The minimum rectangle. (b) Centroid shape.

The different steps of the proposed algorithm for shape representation and recognition will be explained in the following sections and are summarized in Fig. 2.

## 3 B-SPLINE MODELLING

It is assumed that the contour shape is extracted and represented by a set of ordered points. Our goal is to

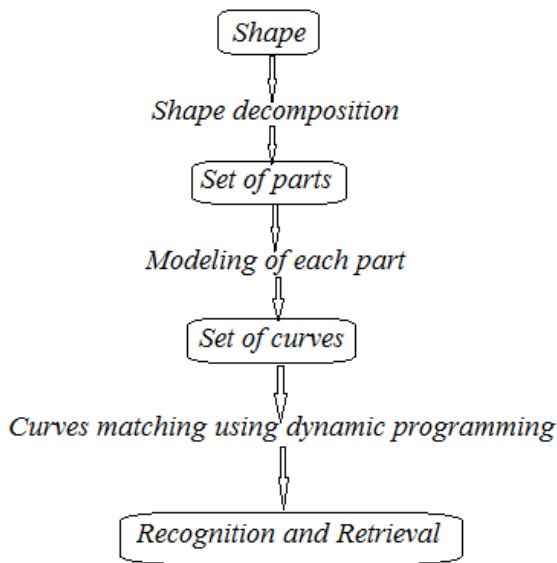


Figure 2: A bloc- diagram of the proposed algorithm for shape representation and retrieval.

give a rough description of the shape using the B-spline model.

First, we have to locate the partitioning points allowing us to decompose shape.

Second, we have to approximate each part by a normalized curve.

### 3.1 Shape Decomposition

Curvature points play an important role in shape representation, reflecting the concave and convex parts of a shape. There are various methods for locating curvature points such as Chetverikov algorithm (Chetvericov, 2003). In our paper, the curvature points are extracted using Peucker Douglas algorithm, which will be used in decomposition process.

For a given boundary shape  $S$  represented by an ordered points, split it into  $n$  different parts:  $S = \bigcup_{i=1}^n S_i$ .  $S_i$  is called a part of  $S$ . To determine the number of parts, we select the concave points from the extracted curvature points.

Only concave points having a high degree of concavity are selected to segment the shape boundary into a set of convex parts (see Fig. 3(b)). The concavity degree  $r/d$  of a point  $p$  is computed as the ratio of the distance  $r$  from  $p$  to associated chord of length  $d$  (Fig. 3(a)).

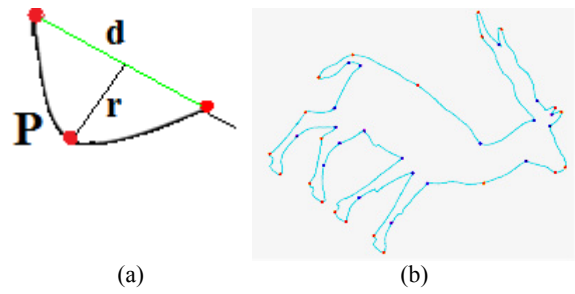


Figure 3: (a) Concavity degree. (b) Decomposition of deer.

### 3.2 Curves Construction

Each part  $S_i$  is approximated by a parametric curve using the B-spline model. This curve is defined relatively to the coordinates system attached to the minimum rectangle enclosing the shape. A B-spline curve is used because B-spline has important properties such as smoothness, continuity and their local control. In our approach a cubic B-spline is chosen instead of a higher order because it is less wiggly.

The B-spline curve  $S(t)$  of order 3 is defined by:

$$S(t) = \sum_{i=0}^m N_{i,3}(t)P_i \quad (4)$$

Where  $N_{i,3}(t)$  is the splines basis functions of order 3 in the parameter  $t \in [0,1]$  (Cohen et al., 1995) and  $P_i = (x_i, y_i); i = 0$  to  $m$  are the B-splines coefficients (control points).

The conventional method to estimate the control points uses an iterative process for adjusting the number of the control points to maintain an error bound (Wang et al., 2006). In our work, the control points are chosen as the high curvature points.

### 3.3 Curves Normalization

In order to simplify comparison of curve shapes, we normalize the measured B-splines curves without changing the shape.

A curve translation procedure is used as the first step of normalization. The centroid of the shape is normalized so as to coincide with the origin (see Fig. 4):

$$x' = x - x_G, y' = y - y_G \quad (5)$$

Where  $(x, y)$  represents a curve point and  $(x', y')$  the corresponding normalized curve point.

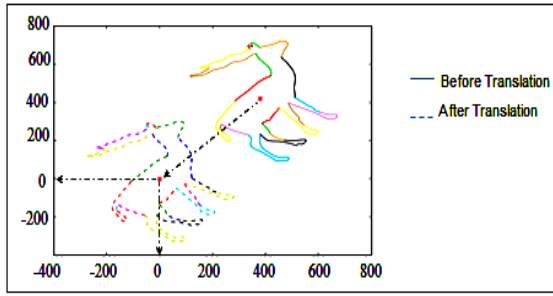


Figure 4: Curve shape translation.

In the second step of normalization, we have to eliminate scaling transformation based on the geometric moments of a shape of an object. There are several ways to normalize the shape size. Adjusting the dimensions of the minimum rectangle, width and high or its area. However in our approach, moments of order up to two are used instead of area because when small deformations occur, area can be altered.

The transformation carried out on the curve points  $(x, y)$  obtained through the B-spline model is defined by

$$x'' = x' / k_1, \quad y'' = y' / k_2 \quad (6)$$

Where the scaling factors  $k_1$  and  $k_2$  represent the normalized moment of order two:

$$k_1 = \sqrt{\frac{M_{20}}{L}} \quad \text{and} \quad k_2 = \sqrt{\frac{M_{02}}{L}} \quad (7)$$

With  $L$  is the length of the B-spline curve.

## 4 SHAPE MATCHING

This section describes a dynamic-programming for establishing correspondences between normalized curves of two shapes. Dynamic programming is an appropriate method for finding associations between segments. It has been used for deformable-template-based segmentation (McNeill and Vijayakumar, 2006). The idea behind our matching process is that if two shapes match, then they share some similar curves. However, it seems appropriate to apply the dynamic programming to establish the best matching pair of curves by using a suitable distance measure.

### 4.1 Similarity between Curves

Measuring the similarity between curves is a key element in object recognition. There are several distances to measure resemblance. Frechet and

Euclidean distance are used for boundary based approaches. In our approach, the Hausdorff distances are used for matching two different curves.

Given two normalized curves  $C$  and  $C'$  of a query shape  $Q$  and a reference shape  $M$  respectively, the Hausdorff distance is defined as:

$$\delta(C, C') = \max(h(C, C'), h(C', C)) \quad (8)$$

Where

$$h(c, c') = \max_{c \in C} \min_{c' \in C'} \|c - c'\| \quad (9)$$

and  $\|\cdot\|$  is a norm defined on the curve, such as the  $L_2$  norm.

This similarity measurement is used for dynamic programming for curve-based shape matching.

### 4.2 Matching using Dynamic Programming

Our aim is to find the best match between a given shape and the query shape by matching their different curves. For this, we use the Dynamic programming (DP). The proposed algorithm tries to build a DP table of costs of partial matches in order to find the minimum cost with the two shapes.

Given two shapes  $Q$  (query shape) and  $M$  (model shape), the DP table has  $q$  rows and  $m$  columns, where  $q$  and  $m$  correspond to the B-splines curves of  $Q$  and  $M$  respectively.

The dynamic programming algorithm can be defined as follows:

Let  $D(C_i, C_j)$  denotes the optimal cost of matching  $C_1, \dots, C_i$ , the first  $i$  curves of  $Q$  with  $C_1, \dots, C_j$ , the first  $j$  curves of  $M$ . It can be defined as:

$$D(C_i, C_j) = \min \begin{cases} D(C_i, C_{j-1}) + \delta(C_i, C_j), \\ D(C_{i-1}, C_j) + \delta(C_i, C_j), \\ D(C_{i-1}, C_{j-1}) + \delta(C_i, C_j). \end{cases} \quad (10)$$

Where  $\delta(C_i, C_j)$  denotes the cost of matching curves  $C_i$  and  $C_j$  defined by the Eq. (8).

Two shapes are considered similar when the cost with both shape representations is minimal.

## 5 EXPERIMENTAL RESULTS

Several experiments have been carried out to test the effectiveness of the proposed approach.

The well known MPEG-7 database (Latecki et al, 2000) is used in our tests. The database contains 1400 images from 70 classes with 20 images per class (see Fig. 5).



Figure 5: Some examples from MPEG-7 database.

The first experiment illustrates some retrieval results from different classes of MPEG-7 database. Each shape has been matched against all the shapes in the database and itself. The obtained results of matching have been ranked using the minimal cost given by the dynamic programming that reflects the similarity between the different normalized curves. For each query, the first twenty closest shapes are shown in Fig. 5.

The queries shapes are in the first row (at the left of each row). The twenty top similar shapes that have been matched by the proposed algorithm are shown in the rest rows.

In order to evaluate the effectiveness of the matching, we have reported under each query shape the obtained hit rate(see Fig. 7). This hit rate is defined as the ratio of the number of retrieved shapes belonging to a certain class to the number of shapes in that class.

A qualitative analysis of the retrieval results is performed.

The retrieval results for the query shape of deer-5

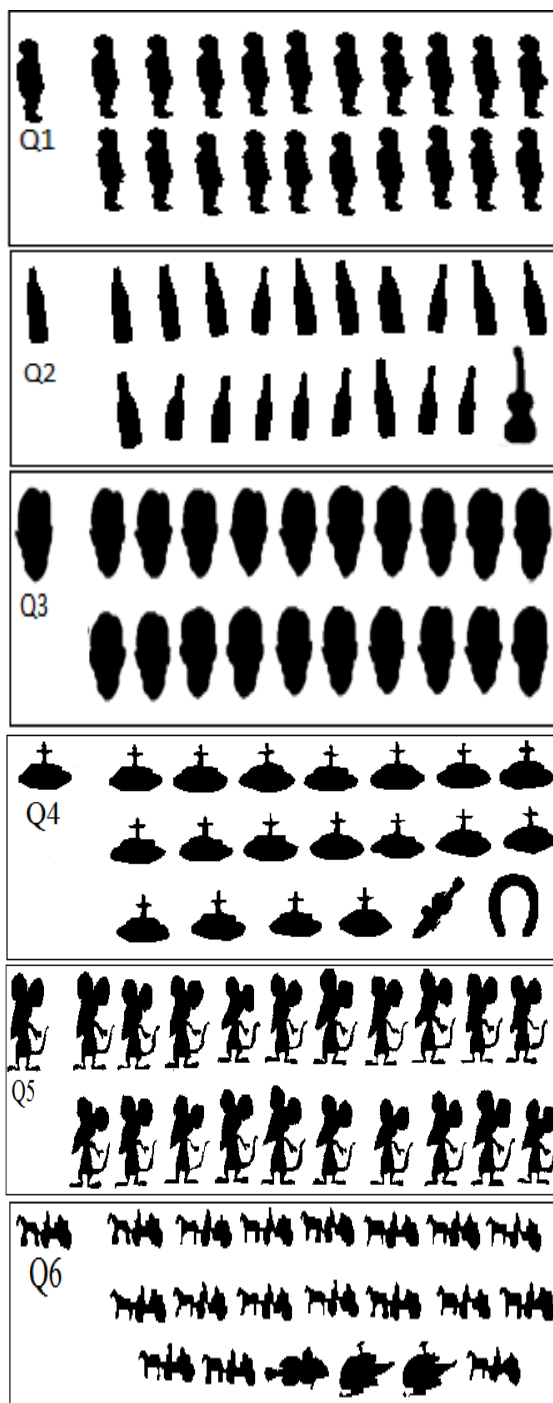


Figure 6: Query shapes and their retrieval results from MPEG-7 database (Latecki et al, 2000). Left column shows query shapes and the right rows show the first 20 ranked nearest neighbours for each query shape.

are compared with the results produced in (Qi et al., 2010) using the methods developed by Wei in (Wei et al., 2009) and the weight-based method in (Jain

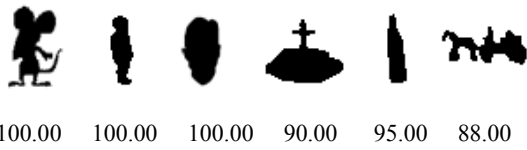


Figure 7: Some query shapes with their recognition rates.

and Vailaya, 1998).

These methods are based on extracting global features such as moments, centroid distances, Zenike moments and edge directions.

Query	Retrieval Results
	<p>(a)</p>
	<p>(b)</p>
	<p>(c)</p>

Figure 8: Top ten retrieval results of shape deer-5 using the weight-based (a). (b) The two-component solution. (c) The proposed approach.

As we can see, the retrieval results of the method proposed by Qi et al. and illustrated by Fig. 8(b) provide 50% precision rate for the top ten retrieval. However, the method of Jain and Vailaya achieve 60% and the proposed approach with 70%.

### 5.1 Dealing with Occlusion

The goal of the second test is to show the robustness of the proposed approach to deal with occluded shapes. For this, we have used shapes of Kimia-99

database (Sebastian et al., 2004) (see Fig. 9). This database consists of nine categories with eleven shapes per category.

In this experiment, we retrieve the top 15 most similar for each query.

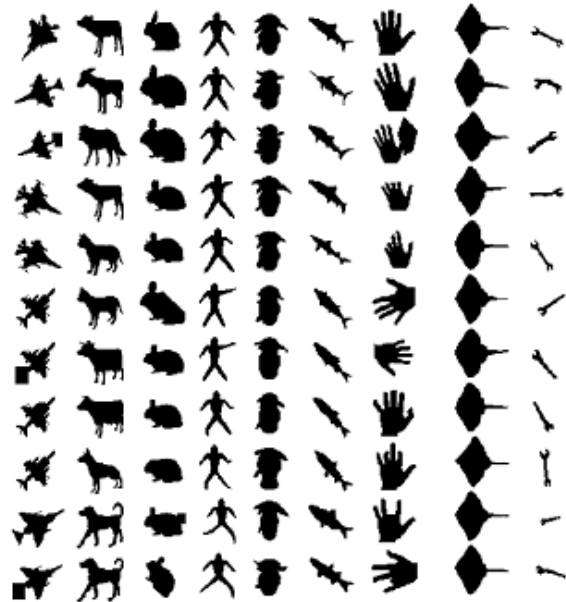


Figure 9: Kimia-99 database.

Table 1 shows an example of this retrieval. The left column represents the query shape. As each class contains 11 shapes, this figure shows that in most cases, most of the shapes from the query class are among the first 11 retrieved shapes.


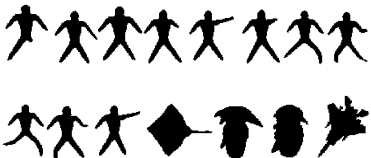

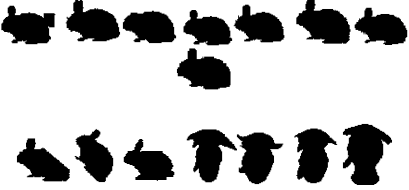



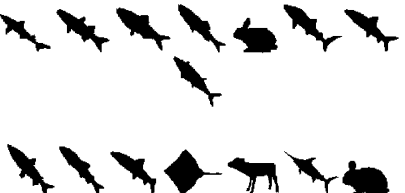
## 6 CONCLUSIONS

In this paper, we have presented a new approach for shape representation based on the B-spline model and dynamic programming. A boundary shape is represented as a sequence of normalized B-splines curves of its meaningful parts. These parts are obtained using curvature points.

A key characteristic of our approach is that describes the different parts constituting the outer closed boundary of the shape. This can be used directly in matching process using the Hausdorff distance and dynamic programming.

The obtained results show the robustness of the approach to several kinds of geometric transformations and occlusion.

Table 1: A tabulation of the top 15 matches for some occluded query shapes.

Query	Retrieval results
	
	
	
	

**REFERENCES**

Alajlan, N., El Rube, I., Kamel, M. S., Freeman, G., 2007. Shape retrieval using triangle-area representation and dynamic space warping. In *Pattern Recognition*, 40, 1911-1920.

Arkin, E., Chew, L., Huttenlocher, D., Mitchell, J., 1991. An efficient computable metric for comparing polygonal shapes. In *IEEE Transactions on Pattern Analysis and Machine Intelligence*. PAMI 13 (3), 209-216.

Belongie, S., Malik, J., Puzicha, J., 2002. Shape matching and object recognition using shape contexts. In *IEEE TRANS. PAMI* 24 (24), 509-521.

Blum, H., 1967. A transformation for extracting new descriptors of shape. In *Models for the perception of speech and visual form*. MIT Press, 362-379.

Carmona-poyato, A., Madrid-Cuevas, F. J., Medina-Carnicer, R., Munoz-Salinas, R., 2010. Polygonal approximation of digital planar curves through break point suppression. In: *Pattern Recognition* 43, 14-25.

Chetvericov, D., 2003. A Simple and efficient algorithm for detection of high curvature points in planar curves. In *10<sup>th</sup> International Conference. CAIP*, 25-27.

Chong, C. W., Raveendran, P., Mukundan, R., 2003. Translation invariants of Zernike moments. In: *Pattern Recognition*. 1765-1773.

Cohen, F. S., Huang, Z., Yang, Z. Invariant matching and identification of curves using B-spline curve representation. In *IEEE Transactions on Image Processing*. 4(1) (1995) 1-10.

Daliri, M. R., Torre, V., 2010. Classification of silhouettes using contour fragments. In *Computer Vision and Image Understanding*. 113, 1017-1025.

Dao, M. S., Amicis, R., 2006. A new method for boundary-based shape matching and retrieval. In: *Proceedings of the International Conference on Image Processing*, 8-11.

Dubois, S. R., Glanz, FH., 1986. An autoregressive model approach to two dimensional shape classification. In: *IEEE Trans Pattern Anal Mach Intell*, 8, 55-65.

Hwang, S. K., Kim, W. Y., 2006. A novel approach to the fast computation of Zernike moments. In: *Pattern Recognition*. 39, 2065-2076.

Jain. A. K., Vailaya. A., 1998. Shape-based retrieval: A case study with trademark image databases. In: *Pattern Recognition*, 31 (9), 1369-1390.

Kim. H., Kim. J., 2000. Region-based shape descriptor invariant to rotation, scale and translation, In: *Signal Processing: Image Communication*, 16, 87-93.

Latecki, L. J., Lakamper, R. 2000. Shape similarity measure based on correspondence of visual parts. In: *IEEE TPAMI*. 22 (10), 1182-1190.

McNeill, G., Vijayakumar, S., 2006. Hierarchical procrustes matching for shape retrieval. In: *CVPR: IEEE International Conf on Computer Vision and Pattern Recognition*, pp. 885-894.

Mokhtarian, F., Abbasi, S., Kittler, J. (1996). Efficient and robust retrieval by shape content through curvature scale space, In *International Workshop on Image Databases and Multimedia Search*, pp. 35-42.

Mongkolnam, P., Nukoolkit, C., Dechsakulthorn, T. 2007. Represent image contents using curves and chain code. In: *IAPR Conference on Machine and Vision Applications*, MVA .355-358.

Paglieroni, D., Jain, A.K., 1985. A Control point theory For boundary representation and matching. In: *Proc. ICASSP*. 1851-1854.

Philip, J., Schneider, D., Eberly, H. Geometric tools for computer graphics, Ed. Textbook Binding, 2002.

Preparata, F., Shamos, M., 1985. *Computational Geometry: An introduction*, Springer, Berlin, Germany.

Qi, H., Li, K., Shen, Y., Qu, W., (2010). An effective solution for trademark image retrieval by combining shape description and feature matching. In: *Pattern Recognition*, 43(6) 2017-2027.

- Sebastian, T. B., Klein, P. N., Kimia, B. B., 2004. Recognition of shapes by editing their shock graphs. In *IEEE TRANS. PAMI* 26 (5), 550-571.
- Singh, C., Pooja, 2011. Improving image retrieval using combined features of Hough transform and Zernike moments. In *Elsevier, Optics and Lasers in Engineerin*, 49 (12), 1384-1396.
- Wang, J., Bai, X., You, X., Liu, W., 2012. Shape matching and classification using height functions. In: *Pattern Recognition Letters*, 33, 134-143.
- Wang, W., Pottmann, H., Liu, Y., 2006. Fitting B-spline curves to point clouds by curvature-based squared distance minimization". In *ACM Transactions on Graphics*, 25(2), 214-38.
- Wang, Y., Teo, E. K. 2004. A novel 2D shape matching algorithm based on B-spline modeling. Singapore.
- Wei, C. H., Li, Y., Chau, W. Y., Li, C. T. (2009). Trade-mark image retrieval using synthetic features for describing global shape and interior structure. In: *Pattern Recognition*, 42 (3) 386-394.
- Yang, G. Y., Shu, H. Z., Toumoulin, C., Han, G. N., Luo, L. M., 2006. Efficient Legendre moments computation for grey level images. In: *Pattern Recognition*. 39, 74-80.
- Zhang, D. S. Lu, G? 2002. Generic Fourier for shape-based image retrieval. In ICME: *IEEE. International Conference on Multimedia and Expo*, pp. 425-428.