Nom:............................ Prénom:.........................

Exercice 1 (4)

1°/ $h(t) = k\, x^*(T_0 - t) = k\left(1 - \frac{(T_0 - t)}{T}\right) = k\left(\frac{T - T_0 + t}{T}\right) = k\, t/_T \quad (T_0 = T)$

$\int h(t)^2 dt = 1$

$\Rightarrow \frac{k^2}{T^2} \int_0^T t^2 dt = 1 = k^2/3T^2 \left[t^3\right]_0^T = k^2 T/_3 \Rightarrow k = \sqrt{3}/\sqrt{T} \Rightarrow h(t) = \frac{\sqrt{3}}{T\sqrt{T}}\, t$ 1

2°/ $SNR = \int |x(t)|^2 dt / \sigma^2 = \frac{1}{\sigma^2} \int_0^T (1 - 2t/T + t^2/T)\, dt = T/(\sigma^2 \times 3)$ 0.5

3°/ $R_{yx}(t - T_0) = \int y(\tau)\, x^*(\tau - (t - T_0))\, d\tau = \int \left[a\, x(\tau - T_D) + b(\tau)\right] x(\tau - (t - T_0))\, d\tau$

$= a\, R_{xx}(t - T_0 - T_D) + R_{bx}(t - T_0)$ 0.5

4°/ Max de $R_{yx}(t - T_0) \Rightarrow t' = \cancel{T_0} + T_D \Rightarrow T_D = t' - T_0$ 0.5 0.5

5°/ $h(t)$ s'exprime en fct de $x^*(T_0 - t)$ 1 pt

Exercice 2 (5.5)

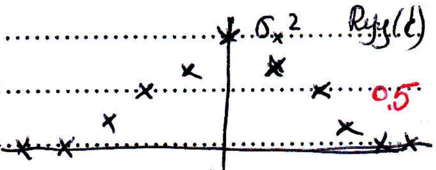
1°/ On suppose que le signal à modeliser $y(n)$ est obtenu par le passage d'un bb dans un filtre $H(f)$ dont on cherche à déterminer les paramètres tels que $S_y(f) = \sigma_b^2 |H(f)|^2$ 0.5 1 pt 0.5

2°/ Sortie en fct juste du bruit $\Rightarrow$ MA 0.5 0.5

3°/ bb $\Rightarrow \mu_x(n) = 0$ 0.5 $\quad \sigma_x^2(n) = \sigma_x^2$ 0.5 $\quad R_x(k) = \sigma_x^2 \delta(k)$ 0.5 $\quad S_x(f) = \sigma_x^2$ 0.5

4°/ $R_{yy}(k) = \sigma^2 \sum_{j=0}^{n-k} b_{j+k}\, b_j \Rightarrow R_{yy}(0) = \sigma_x^2, \quad R_{yy}(1) = 0.75\,\sigma_x^2,$

$R_{yy}(2) = 0.5\,\sigma_x^2 \quad R_{yy}(3) = 0.25\,\sigma_x^2, \quad R_{yy}(k),_4 = 0$ 0.5



$\Rightarrow R_{yy}(k) = \sigma_x^2 \delta(k) + 0.75\,\sigma_x^2\, \delta(k-1) + 0.5\,\sigma_x^2\, \delta(k-2) + 0.25\,\sigma_x^2\, \delta(k-3)$

$\Rightarrow S_{yy}(f) = \sigma_x^2 \left(1 + 0.75\, e^{-2\pi j f T_e} + 0.5\, e^{-4\pi j f T_e} + 0.25\, e^{-6\pi j f T_e}\right)$ 1

on $S_{yy}(f) = \sigma_b^2 |H(f)|^2 = \sigma_b^2 (\cos(\pi f) + \cos 3\pi f)^2$

Exercice 4 (suite)

3°/ $b = 0 \quad \sigma_\theta^2 \xrightarrow[n \to \infty]{} 0 \Rightarrow$ consistant 0.5

4°/ On utilise l'estimateur linéaire à variance minimale pour estimer une v.a x dont on ignore la probabilité mais que l'on a accès aux statistiques d'ordre 2 du signal observé y et de l'intercorrelation de x et y 0.5 pt

Nom:........................ Prénom:........................

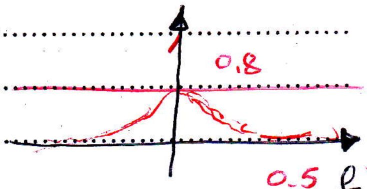
Exercice 3 (6.0)

1% $\mu_s(n) = 0$ $\quad R_{ss}(n) = 0$ $\quad \mu_b(n) = 0$ $\quad b(n)$ bb

0.25 0.25

$$\begin{cases} R_{ss}(k) = 0.8^k \Rightarrow S_{ss}(f) = \frac{z}{z-0.8}\Big|_{z=e^{2\pi f Te}} = \frac{1}{1-0.8e^{2\pi f Te}} \\ R_{bb}(k) = \sigma_b^2\,\delta(k) \Rightarrow S_{bb}(f) = \sigma_b^2 = 0.8 \end{cases}$$

0.5



2% Estimer un signal aléatoire SSL dont le spectre se superpose à celui du bruit

0.5 0.5

3% $$\begin{bmatrix} R_{xx}(0) & R_{xx}(1) & \dots & R_{xx}(N-1) \\ R_{xx}(1) & R_{xx}(0) & \dots & R_{xx}(N-2) \\ \vdots & & & \\ R_{xx}(N-1) & R_{xx}(N-2) & \dots & R_{xx}(0) \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ b_{N-1} \end{bmatrix} = \begin{bmatrix} R_{bx}(0) \\ R_{bx}(1) \\ \vdots \\ R_{bx}(N-1) \end{bmatrix}$$ 0.5

4% $R_{xx}(k) = E\{(s(n)+b(n)-b(n-1))(s(n-k)+b(n-k)-b(n-1-k))\}$

$= R_{ss}(k) + 2R_{bb}(k) - R_{bb}(1+k) - R_{bb}(k-1)$ 0.5

$R_{bx}(k) = E\{b(n)\,x(n-k)\} = E\{b(n)\,[s(n-k)+b(n-k)+b(n-1-k)]\}$

$= R_{bb}(k) - R_{bb}(k+1)$ 0.5

$$\begin{bmatrix} 2.6 & -0.8 \\ -0.8 & 2.6 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \end{bmatrix} = \begin{bmatrix} 0.8 \\ -0.8 \end{bmatrix} \Rightarrow \begin{cases} b_1 = +1/3 \\ b_2 = -1/3 \end{cases}$$ 0.5 0.5

5% $b(n) = +\frac{1}{3}x(n) - \frac{1}{3}x(n-1)$ passe haut 0.5 0.5

Exercice 4 (4.5)

1° $\theta = \frac{1-p}{p} \Rightarrow \theta p = 1-p \Rightarrow p(1+\theta) = 1 \Rightarrow p = \frac{1}{1+\theta}$ 0.5

$\Rightarrow p(x/\theta) = \frac{1}{1+\theta}\left(1-\frac{1}{1+\theta}\right)^x = \frac{\theta^x}{(1+\theta)^{x+1}}$

$\Pi\, p(x_i/\theta) = \frac{\theta^{\Sigma x_i}}{(1+\theta)^{\Sigma x_i + N}}$ $\quad Ln(\Pi\, p(x/\theta)) = \Sigma x_i\, Ln\theta - (\Sigma x_i + N)\, Ln(1+\theta)$

$\frac{d\, Ln(\Pi(p(x_i/\theta)))}{d\theta} = \frac{\Sigma x_i}{\theta} - \frac{\Sigma(x_i+N)}{1+\theta} = 0 \Rightarrow \Sigma x_i + \cancel{\theta\Sigma x_i} = \cancel{\theta\Sigma x_i} + N\theta$

$\Rightarrow \hat{\theta} = \Sigma x_i / N$ 1.5 pt

2% $b = E\{\Sigma x_i/N\} - \theta = \frac{1}{N}\Sigma E\{x_i\} - \theta = \frac{1}{N} N\theta - \theta = 0$

0.5

3% $\sigma^2_{\hat{\theta}} = E\{(\hat{\theta} - E\{\hat{\theta}\})^2\} = E\left\{\left(\frac{\Sigma x_i}{N} - \frac{1}{N}\Sigma\theta\right)^2\right\}$

$= \frac{1}{N^2} E\{(\Sigma(x_i-\theta))^2\}$ puisque x_i indépendants et $E\{x_i-\theta\}=0$

$= 1/N^2\ \Sigma\ \underbrace{E\{(x_i-\theta)^2\}}_{\sigma_x^2} = \frac{\sigma_x^2}{N}$ 1pt