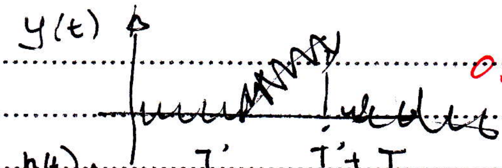
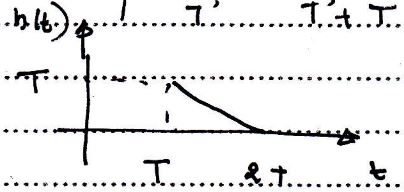
Nom:............................ Prénom:...................

Exercice N° ...1. 4

1° $y(t) = x(t - T') + b(t)$ 0.5

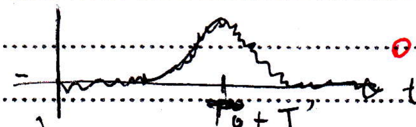


2° $h(t) = \frac{k}{\sigma^2} x^*(T_0 - t) = \frac{k}{\sigma^2}(T_0 - t)$ 0.5



3° $T_0 = 2T$, $k = \sigma^2 \Rightarrow h(t) = 2T - t$

4° $SNR = Ex / \sigma^2 = \int_0^T \frac{t^2 dt}{\sigma^2} = \frac{T^3}{3\sigma^2}$ 0.5



5° Inegalite Cauchy - Swartz ($\leq$) $\Rightarrow$ maximum $\Rightarrow$ optimal bruit blanc $\Rightarrow h(t) = f^{ct}(x(t)) \Rightarrow$ adapté) 0.5

0.5

Exercice N° 2 5.5

1°/ $y(n) = \alpha\, y(n-1) + x(n) \Rightarrow$ ordre 1 avec $a_1 = -\alpha$ 0.5

2°/ On a $\begin{cases} N = 1 \\ k > 0 \end{cases} \Rightarrow R_{yy}(k) = -\sum_{i=1}^{N=1} a_i R_{yy}(k-i) = -(-\alpha) R_{yy}(k-1)$

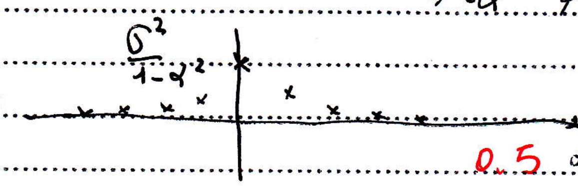
$\Rightarrow R_{yy}(k) = \alpha R_{yy}(k-1) = \alpha^2 R_{yy}(k-2) = \alpha^{|k|} R_{yy}(0)$

$\Rightarrow |\alpha| < 1$ [$R_{yy}(0)$ max] 0.5 1.5 $= R_{yy}(-k)$

3°/ pour $k = 0$ $R_{yy}(0) = \sigma^2 - \sum_{i=1}^{N=1} a_i R_{yy}(i) = \sigma^2 + \alpha R_{yy}(1)$ 1.5

$= \sigma^2 + \alpha^2 R_{yy}(0) \Rightarrow R_{yy}(0) = \frac{\sigma^2}{1-\alpha^2} = \frac{R_{xx}(0)}{1-\alpha^2}$

4°/ - codage parole en téléphonie 0.5
- prédiction météorologique 0.5



0.5 $\alpha > 0$

Exercice N° 3 5.5

1°/ Signal utile et observation SSL R_{yx} et R_{xx} connues
0.25 0.25

2°/ $R_{xx}(k) = E\{x(n).x^*(n-k)\} = E\{[\alpha s(n) + b(n)][\alpha s(n-k) + b(n-k)]\}$
$= \alpha^2 R_{ss}(k) + R_{bb}(k) + \alpha\, \mu s(n)\, \cancel{\mu b(n-k)} + \alpha\, \mu s(n-k)\, \cancel{\mu b(k)}$
1 0 0

$R_{gx}(k) = E\{g(n).x(n-k)\}$
$= E\{s(n)[\alpha s(n-k) + b(n)]\}$
$= \alpha R_{ss}(k) + \mu s\, \cancel{\mu b}$ 0
1

$S_{bb}(f) = 0.25$
$\Rightarrow R_{bb}(k) = 0.25\,\delta(k)$
$\Rightarrow \mu b = 0$
$\sigma_b^2 = 0.25$ 0.5

Nom:............................ Prénom:....................

Exercice N° 3. (Suite)

$R_{xx}(k) = \alpha^2 R_{ss}(k) + R_{bb}(k)$ $\qquad R_{sx}(k) = 2\alpha\, 0.5^{|k|}$

$= 2\alpha^2\, 0.5^{|k|} + 0.25\,\delta(k)$

$$\begin{bmatrix} R_{xx}(0) & R_{xx}(1) \\ R_{xx}(1) & R_{xx}(0) \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \end{bmatrix} = \begin{bmatrix} R_{sx}(0) \\ R_{sx}(1) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2\alpha^2+0.25 & \alpha^2 \\ \alpha^2 & 2\alpha^2+0.25 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \end{bmatrix} = \begin{bmatrix} 2\alpha \\ \alpha \end{bmatrix}$$ 0.5 $\quad Det = (2\alpha^2+0.25)^2 - \alpha^4$

$b_0 = \dfrac{2\alpha(2\alpha^2+0.25) - \alpha^3}{Det}$ 0.5 $\qquad b_1 = \dfrac{\alpha(2\alpha^2+0.25) - 2\alpha^3}{Det}$ 0.5

3°/ $H(z) = b_0 + b_1 z^{-1} \Rightarrow \hat{s}(n) = b_0\, x(n) + b_1\, x(n-1)$

0.5 $\qquad$ 0.5

Exercice N° 4 $\quad$ 5.0

1° Sachant que $p(x) \geq 0\ \forall x \Rightarrow \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}} > 0\ \forall x \Rightarrow x \geq 0$ $\quad$ 1

2° $\prod_{x_i} (p(x_i)) = \dfrac{\prod x_i}{\sigma^{2N}} e^{-\frac{1}{2\sigma^2}\sum_{i=1}^{N} x_i^2}$

$\ln\left[\prod_{i=1\,a\,N} p(x_i)\right] = -\dfrac{1}{2\sigma^2}\sum_{i=1}^{N} x_i^2 - 2N \ln\sigma + \ldots\ldots$

$\dfrac{d \ln\left[\prod_{i=1\,a\,N} p(x_i)\right]}{d\sigma} = +\dfrac{4\sigma}{4\sigma^4}\sum_{i=1}^{N} x_i^2 - \dfrac{2N}{\sigma} = 0 = \dfrac{1}{\sigma^3}\left(\sum_{i=1}^{N} x_i^2 - 2N\sigma^2\right) = 0$

$\Rightarrow \cancel{4\sigma\sum x_i^2 - 4N\sigma^3 = 0} \Rightarrow \hat{\sigma}^2 = \dfrac{1}{2N}\sum_{i=1}^{N} x_i^2$ $\quad$ 2

3° $B_{\hat{\sigma}^2} = E\{\hat{\sigma}^2 - \sigma^2\} = E\{\hat{\sigma}^2\} - \sigma^2 = E\left\{\dfrac{1}{2N}\sum_{i=1}^{N} x_i^2\right\} - \sigma^2$

$= \dfrac{1}{2N}\sum_{i=1}^{N} E\{x_i^2\} - \sigma^2 = \dfrac{1}{2N}\sum_{i=1}^{N}\{\sigma_x^2 + \mu^2\} - \sigma^2$

$= \dfrac{1}{2N}\sum_{i=1}^{N}\left(2\sigma^2 - \cancel{\dfrac{\pi\sigma^2}{2}} + \sigma^2\cancel{\dfrac{\pi}{2}}\right) - \sigma^2 = \dfrac{1}{\cancel{2N}}\cancel{N}\cancel{2}\sigma^2 - \sigma^2 = 0$ $\quad$ 1.5

4° Lorsque probabilité de la variable à estimer est uniforme

0.5