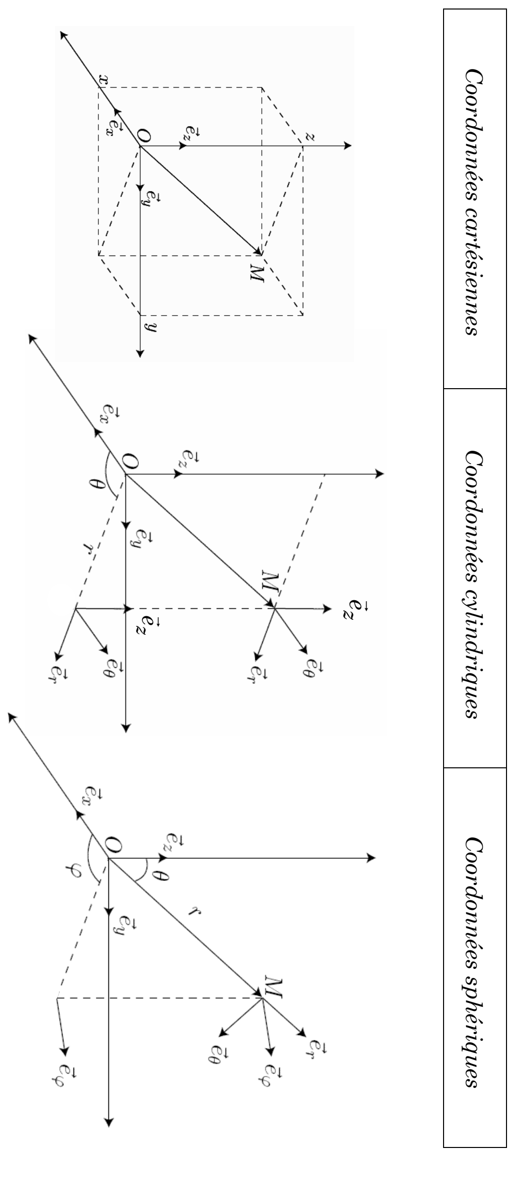
Annexe C

Opérateurs vectoriels en coordonnées cartésiennes, cylindriques et sphériques.



	Coordonnées cartésiennes (x,y,z)	*Coordonnées cylindriques* (r,θ,z)	*Coordonnées sphériques* (r,θ,φ)
$\vec{dl}$	$dx\ \vec{e}_x + dy\ \vec{e}_y + dz\ \vec{e}_z$	$dr\ \vec{e}_r + r\ d\theta\ \vec{e}_\theta + dz\ \vec{e}_z$	$dr\ \vec{e}_r + r\ d\theta\ \vec{e}_\theta + r\sin\theta\ d\varphi\ \vec{e}_\varphi$
$\overrightarrow{grad}\ f(M)$ $\vec{\nabla} f(M)$	$\frac{\partial f}{\partial x}\vec{e}_x + \frac{\partial f}{\partial y}\vec{e}_y + \frac{\partial f}{\partial z}\vec{e}_z$	$\frac{\partial f}{\partial r}\vec{e}_r + \frac{1}{r}\frac{\partial f}{\partial \theta}\vec{e}_\theta + \frac{\partial f}{\partial z}\vec{e}_z$	$\frac{\partial f}{\partial r}\vec{e}_r + \frac{1}{r}\frac{\partial f}{\partial \theta}\vec{e}_\theta + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \varphi}\vec{e}_\varphi$
$div\ \vec{A}(M)$ $\vec{\nabla}\cdot\vec{A}(M)$	$\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$	$\frac{1}{r}\left[\frac{\partial (r A_r)}{\partial r} + \frac{\partial A_\theta}{\partial \theta}\right] + \frac{\partial A_z}{\partial z}$	$\frac{1}{r^2}\frac{\partial (r^2 A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial (A_\theta \sin\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial A_\varphi}{\partial \varphi}$
$\overrightarrow{rot}\ \vec{A}(M)$ $\vec{\nabla}\wedge\vec{A}(M)$	$\left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right]\vec{e}_x + \left[\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right]\vec{e}_y + \left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right]\vec{e}_z$	$\left[\frac{1}{r}\frac{\partial A_z}{\partial \theta} - \frac{\partial A_\theta}{\partial z}\right]\vec{e}_r + \left[\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right]\vec{e}_\theta + \left[\frac{1}{r}\frac{\partial (r A_\theta)}{\partial r} - \frac{1}{r}\frac{\partial A_r}{\partial \theta}\right]\vec{e}_z$	$\frac{1}{r\sin\theta}\left[\frac{\partial (\sin\theta A_\varphi)}{\partial \theta} - \frac{\partial A_\theta}{\partial \varphi}\right]\vec{e}_r + \left[\frac{1}{r\sin\theta}\frac{\partial A_r}{\partial \varphi} - \frac{1}{r}\frac{\partial (r A_\varphi)}{\partial r}\right]\vec{e}_\theta + \frac{1}{r}\left[\frac{\partial (r A_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta}\right]\vec{e}_\varphi$
$\Delta f(M)$ $lap\ f(M)$	$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$	$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 f}{\partial z^2}$	$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial \varphi^2}$

NB:
- Les schémas des coordonnées sont donnés sur la figure ci-contre
- Le vecteur $\overrightarrow{OM}$ est donné par $\overrightarrow{OM} = x\,\vec{e}_x + y\,\vec{e}_y + z\,\vec{e}_z$
- $f(M) = f(x,y,z)$: fonction scalaire de point ou *champ scalaire*
- $A(M) = A_x\,\vec{e}_x + A_y\,\vec{e}_y + A_z\,\vec{e}_z$: fonction vectorielle de point ou *champ vectoriel*