

Chapter IV: Spherical Interfaces & Thin Lenses

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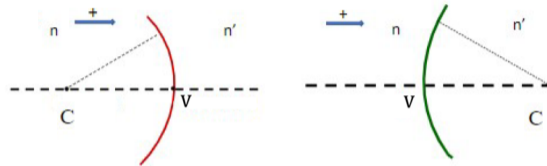
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Outline

- 1 Spherical Interfaces
- 2 Thin Lenses — Definition and Types
- 3 Thin-Lens Equations
- 4 Focal Points and Focal Length
- 5 Transverse Magnification
- 6 Geometrical Construction of Images
- 7 Summary Table
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Definition of a Spherical Interface



Concave: $\overline{VC} < 0$

Convex: $\overline{VC} > 0$

Concave and convex spherical interfaces separating two media of refractive indices n_1 and n_2 .

Definition

A *spherical interface* is the boundary between two transparent, homogeneous, isotropic media of indices n_1 and n_2 . It is a portion of a sphere of radius R .

Key elements

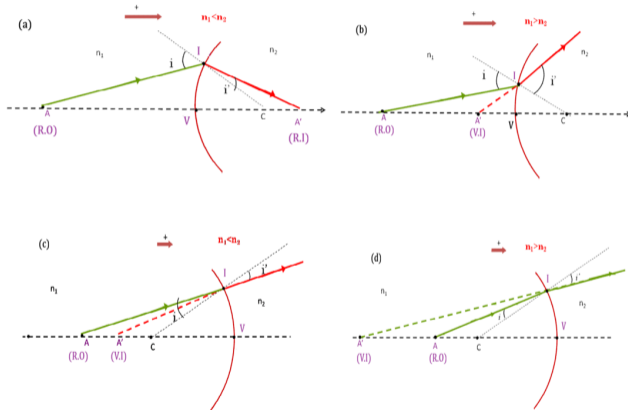
- C : center of curvature,
- V : vertex of the interface,
- $\overline{VC} = R$: algebraic radius of curvature.

Sign convention

Convex: $R > 0$ | Concave: $R < 0$

A **plane interface** is the limiting case $R \rightarrow \infty$.

Image Formation by a Spherical Interface



Four configurations

- (a) Concave interface, $n_1 < n_2$
- (b) Concave interface, $n_1 > n_2$
- (c) Convex interface, $n_1 < n_2$
- (d) Convex interface, $n_1 > n_2$

Remark

Depending on the interface type and on n_1/n_2 , the image A' may be **real or virtual**, closer or farther from V than the object A .

Ray tracing for the four configurations of a spherical interface (vertex V , center C , indices n_1, n_2).

Imaging Equation — Origin at the Vertex

Imaging Equation (Gauss approximation)

$$\frac{n_2}{\overline{VA'}} - \frac{n_1}{\overline{VA}} = \frac{n_2 - n_1}{R}$$

Notation

- \overline{VA} : algebraic object distance,
- $\overline{VA'}$: algebraic image distance,
- $R = \overline{VC}$: radius of curvature.

Derivation principle

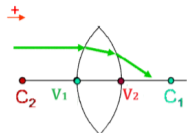
Apply **Snell's law** at the point of incidence M together with the sine rule in the triangles formed by the incident and refracted rays.

Under the *Gauss (paraxial) approximation* ($\sin \theta \approx \theta$), the result is exact and independent of the ray height.

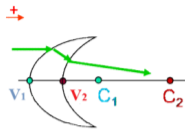
Limiting case: plane interface ($R \rightarrow \infty$)

$$\frac{n_2}{\overline{VA'}} = \frac{n_1}{\overline{VA}} \implies \overline{VA'} = \frac{n_2}{n_1} \overline{VA}$$

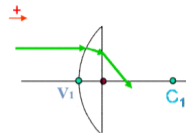
Lenses: Definition and Classification



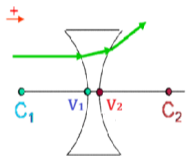
Bi-convex



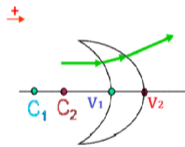
Converging meniscus



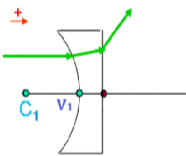
Convex plano



Bi-concave



Diverging meniscus



Concave plano

Definition

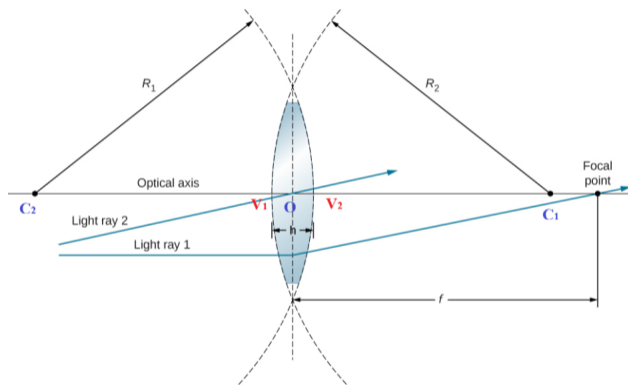
A *lens* is formed by two refracting spherical interfaces (or one spherical + one plane interface) enclosing a transparent medium.

Classification

- **Thin edges** \Rightarrow converging lens ($f' > 0$)
- **Thick edges** \Rightarrow diverging lens ($f' < 0$)

Types: biconvex, plano-convex, converging

Thin Lens Model — Optical Center



Geometrical model of a thin lens: vertices V_1 , V_2 merge into the optical center O .

Thin lens approximation

The thickness of the lens is **negligible** compared with the radii of curvature R_1 , R_2 .

Key definitions

- **Optical center O** : coincidence of V_1 and V_2 .
- **Principal optical axis**: line through O , C_1 , and C_2 .
- Any ray passing through O is **not deviated**.

Gaussian Lens Formula and Power

Thin-lens equation (Gaussian formula)

$$\frac{1}{\overline{OA'}} - \frac{1}{\overline{OA}} = \frac{1}{f'}$$

Obtained by applying the spherical interface equation **twice** (at each face) under the Gauss approximation.

Focal lengths

$$\overline{OF} = f, \quad \overline{OF'} = f', \quad \boxed{f' = -f}$$

Power of a thin lens

$$C = \frac{1}{f'} \quad [\text{diopters, m}^{-1}]$$

- $C > 0$: **converging** lens
- $C < 0$: **diverging** lens

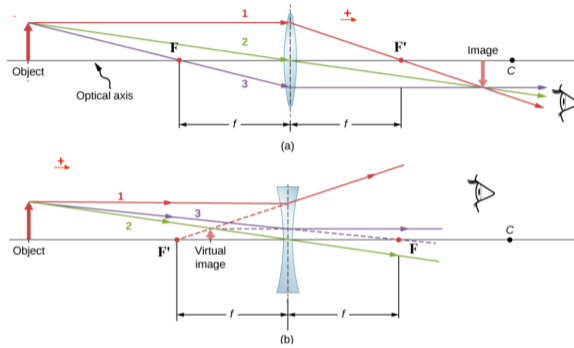
Example

A lens with $f' = +20$ cm:

$$C = \frac{1}{0.20} = +5 \text{ diopters}$$

Converging lens.

Converging and Diverging Lenses — Schematic



Converging lens ($f' > 0$, $C > 0$)

Parallel rays **converge** toward the image focus F' after refraction.

Diverging lens ($f' < 0$, $C < 0$)

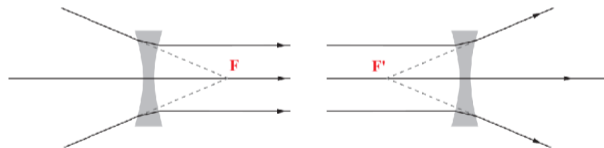
Parallel rays **diverge** after refraction; they appear to come from the virtual image focus F' .

Object Focus F and Image Focus F'

Converging Lens



Diverging Lens



Object and image focal points of converging and diverging thin lenses.

Image focus F'

Point where rays initially **parallel to the axis** converge (or appear to diverge from) after refraction.

$$\overline{OF'} = f'$$

Object focus F

Point from which rays emerge **parallel to the axis** after refraction.

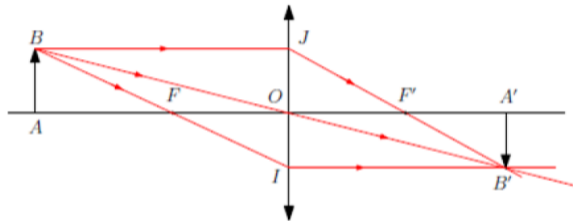
$$\overline{OF} = f = -f'$$

Nature of focal points

Converging lens: F and F' are **real**.

Diverging lens: F and F' are **virtual**.

Transverse Magnification



Conjugate points and transverse magnification M .

Definition

$$M = \frac{\overline{A'B'}}{\overline{AB}} = \frac{\overline{OA'}}{\overline{OA}}$$

Interpretation

- $M > 0$: image is **upright**
- $M < 0$: image is **inverted**
- $|M| > 1$: image is **magnified**
- $|M| < 1$: image is **reduced**

Three Fundamental Construction Rays

Rules for ray tracing through a thin lens

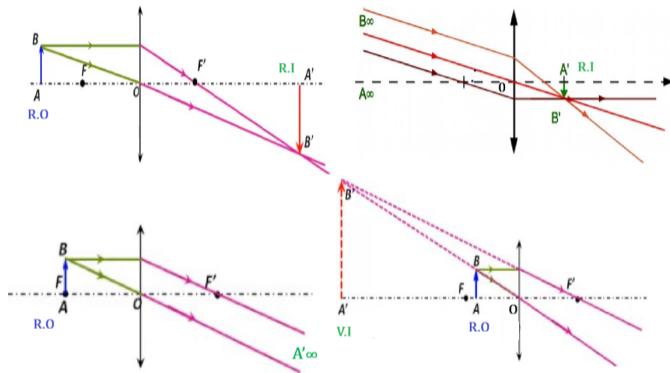
- 1 A ray passing through the **optical center** O is **not deviated**.
- 2 A ray **parallel to the axis** passes through the **image focal point** F' after refraction.
- 3 A ray passing through the **object focal point** F emerges **parallel to the axis**.

General principle

The intersection of **any two** of these rays gives the image point A' . The third ray serves as a **verification**.

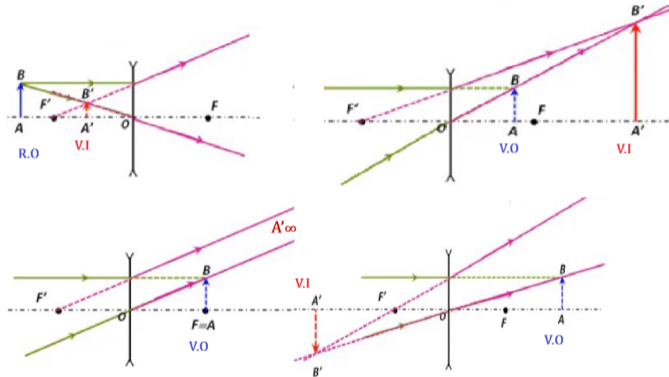
The image nature (real/virtual, upright/inverted, enlarged/reduced) depends on the **position of the object** relative to F and F' .

Image Construction — Converging Lens



Geometrical construction for a converging thin lens: four object positions giving different image configurations (real/virtual, upright/inverted, magnified/reduced).

Image Construction — Diverging Lens



Geometrical construction for a diverging thin lens: four object positions. The image is always virtual, upright, and reduced.

Summary Table — Essential Formulae

Formula	Expression	Meaning
Spherical interface	$\frac{n_2}{\overline{VA'}} - \frac{n_1}{\overline{VA}} = \frac{n_2 - n_1}{R}$	Conjugate relation
Thin-lens equation	$\frac{1}{\overline{OA'}} - \frac{1}{\overline{OA}} = \frac{1}{f'}$	Gaussian formula
Power	$C = 1/f'$	Unit: diopter (m^{-1})
Focal lengths	$f' = -f, \quad \overline{OF'} = f'$	Symmetry relation
Magnification	$M = \overline{A'B'}/\overline{AB} = \overline{OA'}/\overline{OA}$	Size & orientation
Converging lens	$f' > 0, C > 0$	Real F and F'
Diverging lens	$f' < 0, C < 0$	Virtual F and F'

Exercise 1 — Spherical Interface

Given

A spherical convex interface ($R = +8 \text{ cm}$) separates air ($n_1 = 1.00$) from glass ($n_2 = 1.50$). An object is placed at $\overline{VA} = -24 \text{ cm}$.

Questions

- 1 Apply the spherical interface imaging equation to find $\overline{VA'}$.
- 2 Is the image real or virtual?
- 3 What happens if the object is moved to $\overline{VA} = -4 \text{ cm}$?

Numerical answers

$\overline{VA'} = +72 \text{ cm}$ (real image); at $\overline{VA} = -4 \text{ cm}$: $\overline{VA'} = -12 \text{ cm}$ (virtual image).

Exercise 2 — Converging Thin Lens

Given

A converging thin lens has focal length $f' = +15$ cm. An object AB of height $\overline{AB} = 2$ cm is placed at $\overline{OA} = -45$ cm.

Questions

- 1 Calculate $\overline{OA'}$ using the thin-lens equation.
- 2 Determine the transverse magnification M .
- 3 Find the height of the image $\overline{A'B'}$.
- 4 State the nature of the image (real/virtual, upright/inverted).
- 5 What is the power of the lens in diopters?

Numerical answers

$\overline{OA'} = +22.5$ cm; $M = -0.50$; $\overline{A'B'} = -1$ cm; real, inverted, reduced; $P = +6.67$ diopters.

Exercise 3 — Diverging Thin Lens

Given

A diverging thin lens has focal length $f' = -20$ cm. An object is placed at $\overline{OA} = -30$ cm.

Questions

- 1 Calculate the image position \overline{OA}' .
- 2 Determine the magnification M .
- 3 Describe the image (real/virtual, upright/inverted, size).
- 4 Construct the image geometrically using the three fundamental rays.
- 5 Verify your result using $M = \overline{OA}'/\overline{OA}$.

Numerical answers

$\overline{OA}' = -12$ cm; $M = +0.40$; virtual, upright, reduced ($|M| < 1$).

Key takeaways

- The **spherical interface** imaging equation generalises Snell's law to extended objects under the Gauss approximation.
- A **thin lens** combines two interfaces; its behaviour is fully described by the focal length f' (or power P).
- The **Gaussian formula** and **transverse magnification** allow quantitative image analysis.
- Geometrical construction with **three fundamental rays** gives an intuitive picture of the image.

Coming next

Chapter V

Optical Instruments