Chapter III: Prisms

Geometry, Deviation, Minimum Deviation & Emergence Conditions

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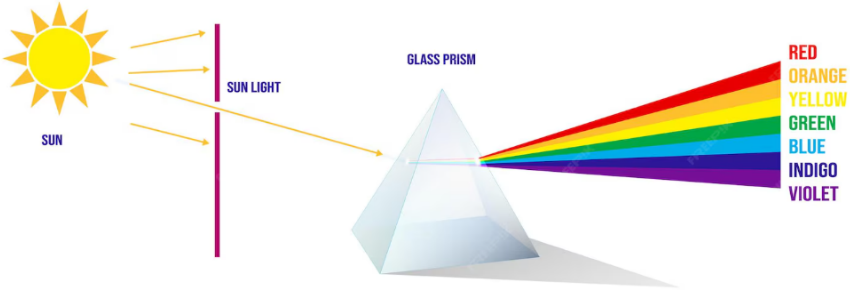
Outline

Dispersion of White Light by a Prism



Observed phenomenon

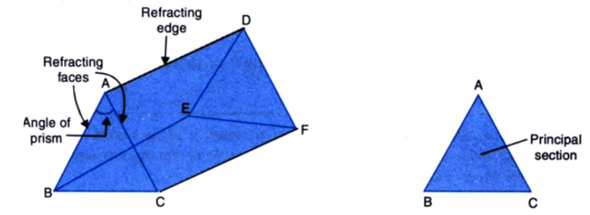
A beam of **white light** enters the prism and emerges decomposed into a colour spectrum: this is **dispersion**.

Physical origin: $n = f(\lambda)$

$n_{\text{violet}} > n_{\text{red}} \Rightarrow$ violet is deviated more than red.

Cauchy's law: $n(\lambda) = a + \dfrac{b}{\lambda^2}$

Prism Geometry — 3D View and Principal Section



3D view (refracting edge, faces, apex angle A) and principal section (triangle ABC).

Key elements

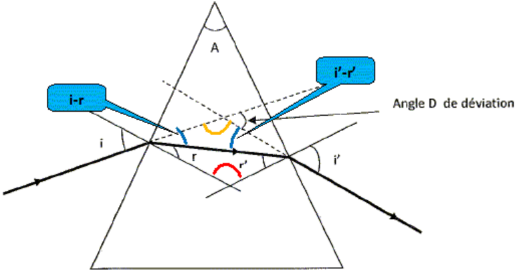
- **Refracting edge**: intersection of the two faces.
- **Apex angle** A: angle between the two faces.
- **Base**: non-refracting face.
- **Principal section**: cross-section $\perp$ to the edge.

Fundamental geometric relation

$$\boxed{r + r' = A}$$

Valid for any ray traversing the prism, independent of n.

Ray Path Through a Prism



Angles i, r (face 1), r' (face 2), i' and total deviation D.

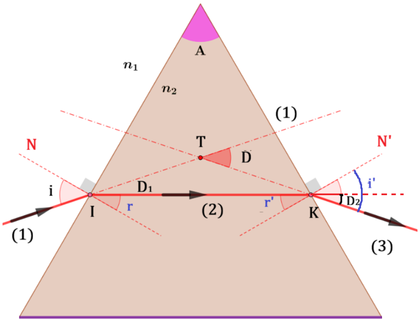
The 4 fundamental equations

1. $\sin i = n \sin r$ (Snell, face 1)
2. $r + r' = A$ (geometry)
3. $n \sin r' = \sin i'$ (Snell, face 2)
4. $\boxed{D = i + i' - A}$ (deviation)

Sequential solution method

$$i \xrightarrow{1} r \xrightarrow{2} r' = A - r \xrightarrow{3} i' \xrightarrow{4} D$$

Decomposition of the Deviation $D = D_1 + D_2$



Partial deviations $D_1 = i - r$ (face 1) and $D_2 = i' - r'$ (face 2).
Total deviation $D = D_1 + D_2$.

Decomposition

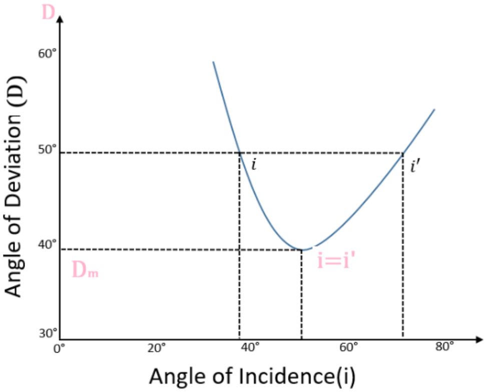
$$D = D_1 + D_2$$

$$D_1 = i - r, \qquad D_2 = i' - r'$$

Algebraic verification

$$\begin{aligned} D &= (i - r) + (i' - r') \\ &= i + i' - (r + r') \\ &= i + i' - A \quad \checkmark \end{aligned}$$

The $D(i)$ Curve and Minimum Deviation D_m



The $D(i)$ curve has a U-shape: unique minimum D_m reached when $i = i'$ (symmetric path).

Properties of the $D(i)$ curve

- For $D > D_m$: **two values** of i are possible.
- Minimum is **unique**: symmetric path.

Conditions at D_m (symmetry)

$$i = i' = i_m, \qquad r = r' = \frac{A}{2}$$

Fundamental prism formula

$$n = \frac{\sin\left(\dfrac{A + D_m}{2}\right)}{\sin\left(\dfrac{A}{2}\right)}$$

Relations and Worked Example — Minimum Deviation

Key relations at D_m

$$D_m = 2i_m - A \implies i_m = \frac{A + D_m}{2}$$

$$\sin i_m = n \sin\left(\frac{A}{2}\right)$$

$$\boxed{n = \frac{\sin\left(\dfrac{A + D_m}{2}\right)}{\sin\left(\dfrac{A}{2}\right)}}$$

Application: $A = 55,\ D_m = 42$

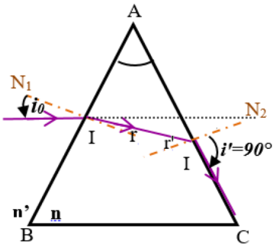
$$n = \frac{\sin(48.5)}{\sin(27.5)} = \frac{0.7490}{0.4617} \approx 1.622$$

Consistent with a **dense optical glass** (e.g. SF2).

Experimental importance

This method is the most accurate for measuring n: only A and D_m need to be measured precisely.

Limiting Emergence Condition — Total Internal Reflection



Limiting case: grazing emergence ($i' = 90$). Beyond this, total internal reflection occurs.

Critical angle r_c

$$\sin r_c = \frac{1}{n}$$

If $r' > r_c$: **total internal reflection**, the ray does not exit the prism.

Condition on the apex angle A

$$A \leq 2r_c = 2\arcsin\left(\frac{1}{n}\right)$$

If $A > 2r_c$: **no** ray can traverse the prism.

Minimum angle of incidence i_0

$$i_0 = \arcsin\left[n\sin(A - r_c)\right]$$

For $i < i_0$: TIR at the second face.

Numerical Example — Emergence Conditions

Crown glass: $n = 1.52$, $A = 60$

$$r_c = \arcsin\left(\frac{1}{1.52}\right) \approx 41.1$$
$$2r_c = 82.2 > A = 60 \quad \Rightarrow \quad \textbf{emergence is possible}$$
$$i_0 = \arcsin\big[1.52\,\sin(60 - 41.1)\big] = \arcsin(0.4923) \approx 29.5$$

Conclusion

Any ray with $i \geq 29.5$ exits the prism.
For $i < 29.5$: TIR at the second face.

Important remark

At minimum deviation: $i_m > i_0$ always holds.
The symmetric configuration always lies within the emergence domain.

Summary Table — Essential Formulae

Formula	**Expression**	**Meaning**
Snell (face 1)	$\sin i = n \sin r$	Entry into prism
Geometry	$r + r' = A$	Interior triangle
Snell (face 2)	$n \sin r' = \sin i'$	Exit from prism
Total deviation	$D = i + i' - A$	Definition of D
Min. deviation	$r = r' = \frac{A}{2}, \quad i = i'$	Symmetric path
Fundamental formula	$n = \dfrac{\sin\left(\frac{A+D_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$	Measurement of n
Critical angle	$\sin r_c = 1/n$	Glass–air interface
Apex condition	$A \leq 2r_c$	Emergence possible
Min. incidence	$i_0 = \arcsin[n \sin(A - r_c)]$	Emergence threshold

Exercise 1 — Fundamental Prism Equations

Given

Glass prism: $n = 1.58$, $A = 48$, ray incident at $i = 37$.

Questions

1. Calculate the refraction angle r at the first face.
2. Determine the internal angle r' at the second face.
3. Calculate the emergence angle i'.
4. Compute the total deviation D.
5. Verify using $D = D_1 + D_2$ where $D_1 = i - r$, $D_2 = i' - r'$.

Numerical answers

$r \approx 22.6$; $r' = 25.4$; $i' \approx 43.7$; $D \approx 32.7$; $D_1 \approx 14.4$, $D_2 \approx 18.3$. ✓

Exercise 2 — Minimum Deviation and Refractive Index

Given

Prism of apex angle $A = 62$, measured minimum deviation $D_m = 48$.

Questions

1. Write the symmetry conditions at minimum deviation.
2. Calculate the angle of incidence i_m.
3. Determine n using the fundamental prism formula.
4. For $n = 1.65$ and the same A: find D_m.
5. Using Cauchy $n(\lambda) = 1.492 + B/\lambda^2$, find B if $n = 1.540$ at $\lambda = 480\,\text{nm}$.

Answers

$i_m = 55$; $n \approx 1.597$; $D_m(n{=}1.65) \approx 56.7$; $B \approx 0.011\,\mu\text{m}^2$.

Exercise 3 — Emergence Conditions

Given

Equilateral prism ($A = 60$), refractive index $n = 1.73$.

Questions

1. Calculate the critical angle r_c.
2. Verify $A \leq 2r_c$ and conclude on emergence.
3. Determine the minimum angle of incidence i_0.
4. Ray at $i = 20$: show that total internal reflection occurs.
5. Ray at $i = 50$: verify emergence and calculate D.

Answers

$r_c \approx 35.3$; $2r_c \approx 70.6 > 60$: emergence possible; $i_0 \approx 46.1$; at $i{=}20$: TIR confirmed; at $i{=}50$: $D \approx 38.2$.

Questions?

Next: Chapter IV - Thin Lenses