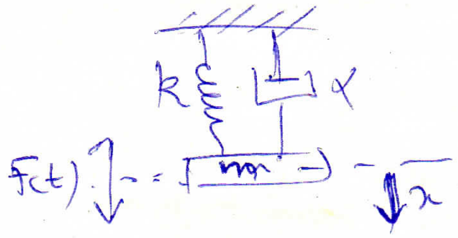
exo



$m = 10\,kg \quad \alpha = 20\,N\,s/m$

$k = 4000\,N/m$

$x_0 = 0{,}01\,m \quad ; \quad \dot{x}_0 = 0.$

? $x(t)$

a) $F(t) = F_0 \cos\omega t$ avec $F_0 = 100\,N$ & $\omega = 10\,rad/s$.

eqt diff de mvt : $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = -\frac{\partial \mathcal{D}}{\partial \dot{x}} + \mathcal{F}_x(t) \Leftarrow$ (I)

où $\mathcal{F}(t)$ = force généralisée $= \vec{F}(t) \cdot \frac{d\vec{r}}{dx}$ → Force : si mvt de translation → Moment : // // de rotation

Dans notre cas : mvt de translation : $\mathcal{F}_x = F(t)$

$L = T - U = \frac{1}{2} m \dot{x}^2 - \frac{1}{2} k x^2$

$\mathcal{D} = \frac{1}{2} \alpha \dot{x}^2$

(I) $\longrightarrow \quad m\ddot{x} + kx = -\alpha \dot{x} + F(t)$

$\Rightarrow \quad m\ddot{x} + \alpha \dot{x} + kx = F_0 \cos\omega t \Leftarrow \boxed{\ddot{x} + 2\delta \dot{x} + \omega_0^2 x = A\cos\omega t}$ (II)

où $\begin{cases} 2\delta = \frac{\alpha}{m} \\ \omega_0^2 = \frac{k}{m} \end{cases}$ et $\boxed{A = \frac{F_0}{m}}$

solution homogène

Solution de (II) $\Rightarrow x(t) = x_H(t) + x_p(t)$

sol. particulière

AN : $\omega_0 = 20\,rad/s$, $\delta = 1\,rd/s$ $\Rightarrow \omega_0 > \delta \Rightarrow$ pseudo-périodique

$\omega_a = 19{,}975\,rad/s$.

$A = 10\,N/kg$

$x_H(t) = B\,e^{-\delta t} \cos(\omega_a t + \Phi)$; $x_p(t) = \Omega \cos(\omega t + \varphi)$

$\dot{x}_H(t) = -\omega_a B e^{-\delta t} \sin(\omega_a t + \Phi) - \delta \cdot x_H(t)$

$\Omega = \frac{A}{\sqrt{(\omega_0^2 - \omega^2)^2 + (2\delta\omega)^2}}$

$\dot{x}_p(t) = -\omega \Omega \sin(\omega t + \varphi)$

$tg\,\varphi = \frac{-2\delta\omega}{\omega_0^2 - \omega^2}$

On applique les conditions initiales

$$\begin{cases} x(t=0) = \cancel{x(0)}\ x_0 \\ \dot{x}(t=0) = \dot{x}_0 \end{cases}$$

(Ds le cas de notre exercice.

et $\begin{cases} x_0 = 0{,}01\,m & : \text{déplacement initial} \\ \dot{x}_0 = 0 & : \text{vitesse nulle} \end{cases}$

$$\begin{cases} x(0) = x_H(0) + x_p(0) = B\cos\Phi + \mathcal{A}\cos\varphi \\ \dot{x}(0) = \dot{x}_H(0) + \dot{x}_p(0) = -\omega_a B\sin\Phi - \delta x_H(0) - \omega\mathcal{A}\sin\varphi \end{cases}$$

(rap: $\mathcal{A}$ et φ sont connues) Calcul:

$$\cancel{B\sin\Phi = -\dot{x}(0) + \delta x_H(0)}$$

$$\mathcal{A} = \frac{\frac{F_0}{m}}{\sqrt{(\omega_0^2-\omega^2)^2+(2\delta\omega)^2}} \Longrightarrow \mathcal{A} = 3{,}34\,.10^{-2}\,m.$$

$$tg\,\varphi = \frac{-2\delta\omega}{\omega_0^2-\omega^2} \Longrightarrow \varphi = -3{,}814^\circ$$

$$x_H(0) = x(0) - x_p(0) = x(0) - \mathcal{A}\cos\varphi \rightarrow \begin{cases} x_H(0) = -2{,}33\,.10^{-2}\,m \\ x_H(0) = B\cos\Phi \end{cases}$$

$$B\sin\Phi = \left[\dot{x}(0) + \delta.x_H(0) + \omega\mathcal{A}\sin\varphi\right]\frac{1}{(-\omega_a)} \longrightarrow B\sin\Phi = 2{,}279\,.10^{-3}\,m$$

$$\rightarrow B = 2{,}34\,.10^{-2}\,m.\ \&\ \Phi = -5{,}586^\circ$$

$$\text{D'où}\quad x(t) = Be^{-\delta t}\cos(\omega_a t + \Phi) + \mathcal{A}\cos(\omega t + \varphi)$$

avec

$B = 2{,}34\,.10^{-2}\,m.$ $\qquad \mathcal{A} = 3{,}34\,.10^{-2}\,m$

$\delta = 1$ rad/s $\qquad \omega = 10$ rad/s

$\omega_a = 19{,}975$ rad/s $\qquad \varphi = -3{,}814^\circ$

$\Phi = -5{,}586^\circ$

b/. $F(t) = 0$ Mvt amorti.

$$\ddot{x} + 2\delta \dot{x} + \omega_0^2 x = 0$$

$$x(t) = B e^{-\delta t} \cos(\omega_a t + \Phi)$$

$\delta = 1$ rad/s

$\omega_a = 19{,}975$ rad/s

$$\begin{cases} x(0) = 0{,}1\,m \\ \dot{x}(0) = 0 \end{cases} \Rightarrow \begin{cases} B\cos\Phi = 0{,}1 & (1) \\ -\omega_a B \sin\Phi - \delta B \cos\Phi = 0 & (2) \end{cases}$$

$(2) \Rightarrow \operatorname{tg}\Phi = \frac{-\delta}{\omega_a} \Rightarrow \boxed{\Phi = -2{,}866^\circ}$

(2) et $(1) \Rightarrow \boxed{B = 10{,}012 \cdot 10^{-2}\,m}$