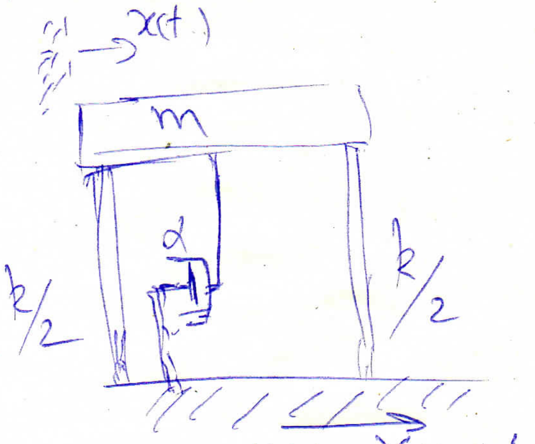
exo.



$y(t) = Y\cos\omega t$

$y = Y\cos\omega t$
$\dot y = -\omega Y\sin\omega t$
$\ddot y = -\omega^2 Y\cos\omega t$

on pose $z = x - y$
$\dot z = \dot x - \dot y$
$\ddot z = \ddot x - \ddot y$

$T = \frac{1}{2} m \dot x^2$

$U = \frac{1}{2} k (x-y)^2$ eq de mvt: $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot x}\right) - \frac{\partial L}{\partial x} = -\frac{\partial \mathcal{D}}{\partial \dot x}$

$\mathcal{D} = \frac{1}{2}\alpha(\dot x - \dot y)^2$

$\Rightarrow m\ddot x + k(x-y) = -\alpha(\dot x - \dot y)$

$\Rightarrow m(\ddot z + \ddot y) + kz + \alpha \dot z = 0$

$\Leftrightarrow m\ddot z + \alpha \dot z + kz = -m\ddot y = m\omega^2 Y\cos\omega t$

$\Leftrightarrow \ddot z + 2\delta \dot z + \omega_0^2 z = \omega^2 Y\cos\omega t$ $\begin{cases} \delta = \frac{\alpha}{2m} \\ \omega_0^2 = \frac{k}{m} \end{cases}$

Solution en régime permanent.

$z = Z\cos(\omega t + \varphi)$

$Z = \frac{\omega^2 Y}{\left[(\omega_0^2 - \omega^2)^2 + (2\delta\omega)^2\right]^{1/2}}$ et $\operatorname{tg}\varphi = \frac{-2\delta\omega}{\omega_0^2 - \omega^2}$

l'énergie dissipée sur un cycle (une période) par l'amortisseur est donnée par: $\left(\frac{dE}{dt} = f_\alpha \cdot v_\alpha\right)$

$E = \pi.\alpha.\omega.Z^2$; $E = \int_0^{T=\frac{2\pi}{\omega}} dE = \int_0^T -\alpha \frac{dz}{dt} \cdot \frac{dz}{dt} \cdot dt$

$= \int_0^{t=T} \alpha \left(\frac{dz}{dt}\right)^2 dt$

$E = \frac{\pi.\alpha.\omega.(\omega^2.Y)^2}{(\omega_0^2 - \omega^2)^2 + (2\delta\omega)^2}$

E devenue fonction de α ; $\delta = \dfrac{\alpha}{2m}$

E est max ssi $\dfrac{dE}{d\alpha} = 0$

$$\frac{d\delta}{d\alpha} = \frac{1}{2m}$$

$$\frac{dE}{d\alpha} = \frac{dE}{d\delta} \cdot \frac{d\delta}{d\alpha}$$

$$= \frac{1}{2m} \cdot \frac{dE}{d\delta} = 0 \Leftrightarrow \frac{dE}{d\delta} = 0$$

$$E = \frac{\pi \alpha . \omega \left(\omega^2 \gamma\right)^2}{\left(\omega_0^2 - \omega^2\right)^2 + \left(2\delta\omega\right)^2}$$

$$= \frac{2m\pi . \omega^5 . \gamma . \delta}{\left(\omega_0^2 - \omega^2\right)^2 + 4\omega^2 . \delta^2}$$

$$\frac{dE}{d\delta} = \frac{2m\pi\omega^5 \gamma \times \left[\left(\omega_0^2-\omega^2\right)^2 + 4\omega^2\delta^2\right] - 2m\pi\omega^5\gamma\delta . 8\omega^2\delta}{\left[\left(\omega_0^2-\omega^2\right)^2 + \left(2\delta\omega\right)^2\right]^2} = 0$$

$$\Leftrightarrow \left(\omega_0^2 - \omega^2\right)^2 + 4\omega^2\delta^2 - 8\omega^2\delta^2 = 0$$

$$\Leftrightarrow \delta^2 = \frac{\left(\omega_0^2 - \omega^2\right)^2}{4\omega^2}$$

$$\delta = \sqrt{\frac{\left(\omega_0^2 - \omega^2\right)^2}{4\omega^2}} \Leftrightarrow \boxed{\alpha = 2m \cdot \left[\frac{\left(\omega_0^2 - \omega^2\right)^2}{4\omega^2}\right]^{1/2}}$$

$$\boxed{\alpha = \frac{m\left(\omega_0^2 - \omega^2\right)}{\omega} = \frac{k - m\omega^2}{\omega}}$$

l'amortisseur qui dissipe le maximum d'énergie