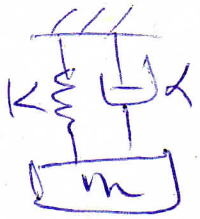
exo : ? ξ et ω_a avec $\xi = \dfrac{\alpha}{2m\,\omega_0} = \dfrac{\delta}{\omega_0}$



; $\omega_0^2 = \dfrac{k}{m}$; $\delta = \dfrac{\alpha}{2m}$ $\quad \omega_a = \sqrt{\omega_0^2 - \delta^2}$

$= \omega_0\sqrt{1-\xi^2}$

ⓐ : $\begin{cases} m = 10\ kg \\ \alpha = 150\ Ns/m \\ k = 1000\ N/m \end{cases} \Rightarrow \begin{cases} \omega_0 = 10\ rad/s \\ \xi = 0{,}75 \\ \omega_a = 6{,}614\ rd/s > 0 \end{cases}$ ⇒ mvt pseudopériod

ⓑ $\begin{cases} m = 10\ kg \\ \alpha = 200\ Ns/m \\ k = 1000\ N/m \end{cases} \Rightarrow \begin{cases} \omega_0 = 10\ rad/s \\ \xi = 1{,}0 \\ \omega_a = 0 \end{cases}$ ⇒ régime critique

ⓒ $\begin{cases} m = 10\ kg \\ \alpha = 250\ Ns/m \\ k = 1000\ N/m \end{cases} \Rightarrow \begin{cases} \omega_0 = 10\ rad/s \\ \xi = 1{,}25 \\ \omega_a : \text{Non calculable car devient Complexe} \end{cases}$ ⇒ mvt Apériodique

$\begin{cases} \xi < 1 \Leftrightarrow \dfrac{\delta}{\omega_0} < 1 \Rightarrow \delta < \omega_0 \Rightarrow \text{mvt périodique} \\ \xi = 1 \Leftrightarrow \dfrac{\delta}{\omega_0} = 0 \Rightarrow \delta = \omega_0 \Rightarrow \text{'' critique.} \\ \xi > 1 \Leftrightarrow \dfrac{\delta}{\omega_0} > 1 \Rightarrow \delta > \omega_0 \Rightarrow \text{'' Apériodique} \end{cases}$

(a) mvt pseudopériodique:

$$\delta = \xi . \omega_0$$

$$\begin{cases} x(t) = A e^{-\delta t} \sin(\omega_a t + \varphi) \\ \dot{x}(t) = -\delta . x(t) + \omega_a . A e^{-\delta t} \cos(\omega_a t + \varphi) \end{cases}$$

$x(0) = x_0 = 0,1 \, m$ et $\dot{x}(0) = \dot{x}_0 = 10 \, m/s$

$$\begin{cases} x(0) = A \sin\varphi \\ \dot{x}(0) = -\delta \, x(0) + \omega_a A \cos\varphi \end{cases} \Rightarrow A = \sqrt{x^2(0) + \left(\frac{\dot{x}(0) + \delta . x(0)}{\omega_a}\right)^2}$$

$$\Rightarrow tg\,\varphi = \omega_a . \frac{x(0)}{(\dot{x}(0) + \delta \, x(0))}$$

(b) mvt critique:

$$\begin{cases} x(t) = (A_1 t + A_2) e^{-\delta t} \\ \dot{x}(t) = A_1 e^{-\delta t} - \delta . x(t) \end{cases} ; \; \delta = \omega_0 .$$

$$\begin{cases} \boxed{x(0) = A_2} \\ \dot{x}(0) = A_1 - \delta . x(0) \end{cases} \Rightarrow \boxed{A_1 = \dot{x}(0) + \delta . x(0)}$$

(c) mvt Apériodique:

$$x(t) = C_1 e^{r_1 t} + C_2 e^{r_2 t}$$

r_1 & r_2 racines de l'eqt caractéristique